# CHAPTER THIRTEEN

## DC DRIVES USING CONTROLLED RECTIFIERS

### 13.1 INTRODUCTION

The speed of d.c. motor can be controlled very easily by means of regulating its supply voltage by the use of phase controlled rectifiers. This control can be applied to either the field or the armature circuit. The motor response with armature control is faster than that with field control since the time constant of the field is very much larger than that of the armature. Generally, field control is used for speeds above rated value and armature control for speeds below rated value.

Controlled rectifiers either single-phase or multi-phase are widely used in d.c. drive applications where a.c. source is available. The single-phase or multi-phase a.c. is converted to d.c. by a controlled rectifier or converter to give a variable d.c. source, by varying the triggering angle of the thyristor or any other power semiconductor device, that could be supplied to a d.c. motor and thus the speed of the motor can be controlled. Controlled rectifiers used in d.c. drives can be classified as follows:

1- Single-phase controlled rectifiers

(i) Single-phase half-wave converter drives.
(ii) Single-phase full-wave half-controlled converter drives.
(iii) Single-phase full-wave fully-controlled converter drives.
(iv) Single-phase dual converter drives.

2. Multi-phase controlled rectifiers

(i) Three-phase half-wave converter drives.
(ii) Three-phase full-wave fully-controlled converter drives.
(iii) Three-phase full-wave half-controlled converter drives.
(iv) Three-phase full-wave dual converter drives.

## 13.2 SINGLE-PHASE CONVERTER DRIVES

These are used for small and medium power motors up to 75kW (100*hp*) ratings. In the following subsections, the various types of these converters will be discussed.

### 13.2.1 Single-Phase Half-Wave Converter Drives

Fig.13.1 shows a single-phase half-wave converter drive used to control the speed of separately-excited motor. This d.c. drive is very simple, needs only one power switch and one freewheeling diode connected across the motor terminals for the purpose of dissipation of energy stored in the inductance of the motor and to provide an alternative path for the motor current to allow the power switch to commutate easily.

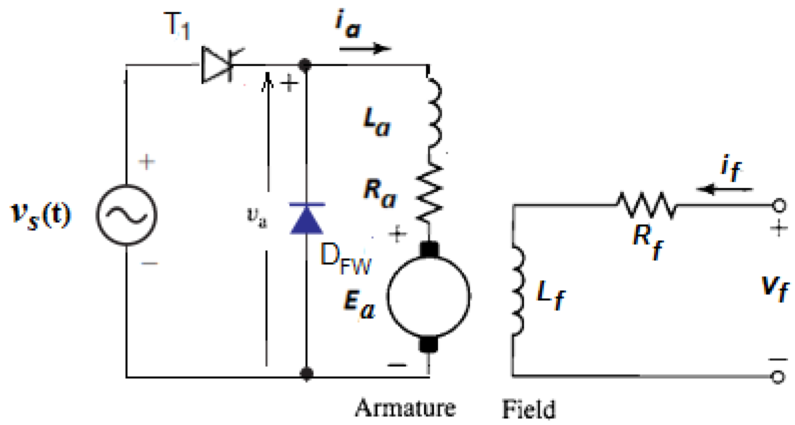



Fig.13.1 Single-phase half-wave converter drive.

Waveforms for steady-state operation of the converter with motor load is shown in Fig.13.2 for the case $\alpha \equiv 80^\circ$. It is clear that during the interval $\beta < \omega t < 2\pi$ , the armature current is zero, hence the torque developed by the motor is zero, the speed of the motor will be reduced. Since the mechanical time constant of the motor is larger than its electrical time constant, the inertia of the motor will maintain the speed, but its value will fluctuate resulting in poor motor performance. Therefore, this type of drive is rarely used; it is only used for small d.c. motors below 500 W ratings.

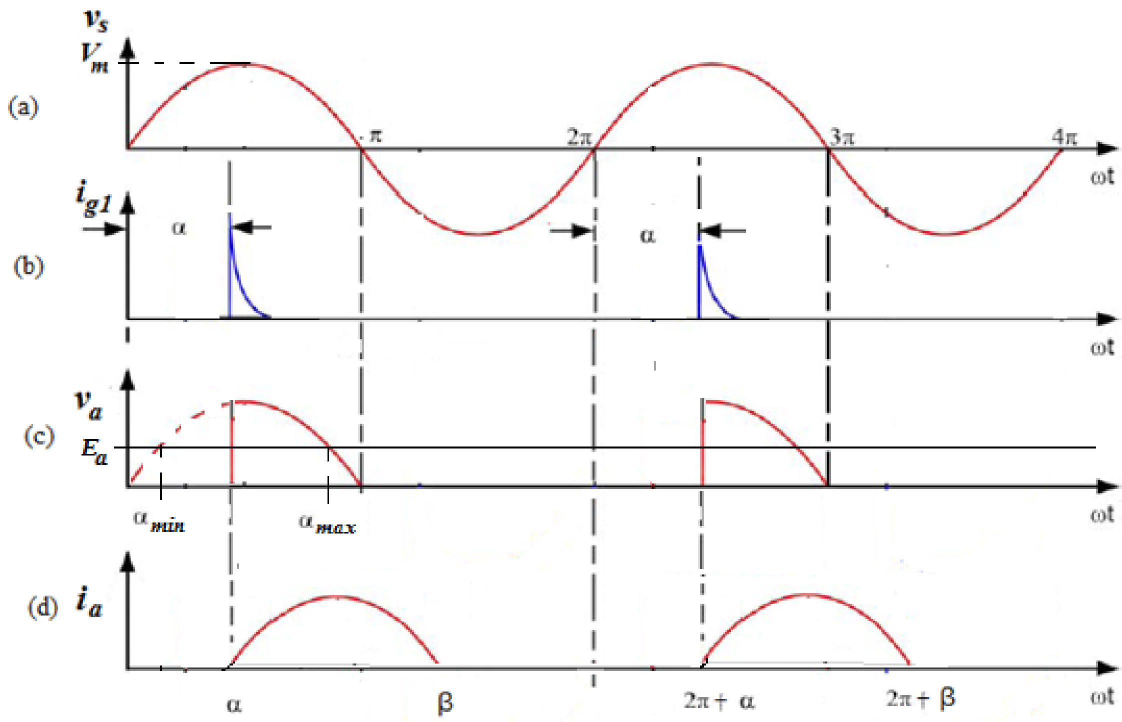



Fig.13.2 Waveforms for steady-state operation of the single-phase half-wave rectifier with motor load.

The average value of the armature voltage can be evaluated as follows:

Assuming the supply voltage $v_s(\omega t) = V_m \sin \omega t$, thus in the positive half-cycle, $T_1$ will conduct from α to π , where α is the firing angle, and $D_{FW}$ will conduct from π to $\beta$, where $\beta$ is the extinction angle of the current. Hence the average value of the armature current will be,

$$V_{a(av)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_s(\omega t)\ d\omega t\ = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin \omega t\ d\omega t$$
$$= \frac{V_m}{2\pi}\,(1+\cos\alpha) \qquad (13.1)$$

It is to be noted that the thyristor $T_1$ is only conducts when supply voltage exceeds back *emf* $E_a$ , therefore, referring to Fig.13.2(c), we define two triggering angles $\alpha_{min}$ and $\alpha_{max}$ as,

$\alpha_{min}$ is the minimum firing angle below which the thyristor cannot be triggered. i.e. when the supply voltage $V_m\, sin\alpha\ > E_a$. This angle can be calculated as,

$$V_m \sin \alpha_{min} = E_a \qquad \rightarrow \qquad \alpha_{min} = sin^{-1}\left(\frac{E_a}{V_m}\right) \qquad (13.2)$$

Similarly $\alpha_{max}$ is the maximum firing angle above which the thyristor cannot be triggered. Its value is given by

$$\alpha_{max} = \pi - \alpha_{min} = \pi - sin^{-1}\left(\frac{E_a}{V_m}\right) \qquad (13.3)$$

The speed of the motor can be calculated from the general equation of the speed of d.c. motor as,

$$n = \frac{V_a}{K_e \emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_L$$

Substituting for $V_{a(av)}$ from Eq.(13.1) into the above equation we get,

$$n = \frac{V_m}{2\pi\, K_e\, \emptyset}(1 + \cos\alpha) - \frac{R_a}{K_T K_e \emptyset^2} T_L \qquad (13.4)$$

or in terms of the angular velocity $\omega$ using Eq.(11.25),

$$\omega = \frac{V_{a(av)}}{K\emptyset} - \frac{R_a}{K^2 \emptyset^2} T_L$$

Substituting for $V_{a(av)}$ we get,

$$\omega = \frac{V_m}{2\pi K\emptyset}(1 + \cos\alpha) - \frac{R_a}{K^2 \emptyset^2} T_L \qquad (13.5)$$

The starting torque can also be calculated from Eq.(13.4) or Eq.(13.5) by setting n or ω equal zero and calculate the torque, (using Eq.(13.5) for example), as

$$0 = \frac{V_m}{2\pi K\emptyset}(1 + \cos\alpha) - \frac{R_a}{K^2 \emptyset^2} T_{st}$$

From which,

$$T_{st} = \frac{K\emptyset V_m}{2\pi R_a}(1 + \cos\alpha) \qquad (13.6)$$

And the no load speed is calculated from Eq.(13.5) by setting $T_L = 0$ to give,

$$\omega_o = \frac{V_m}{2\pi K\emptyset}(1 + \cos\alpha) \qquad (13.7)$$

By knowing $\omega_o$ and $T_{st}$ the mechanical characteristics of the motor can be obtained for various values of the triggering angle $\alpha$ as illustrated in the following example:

## Example 13.1

For the d.c. drive circuit of a separately-excited motor shown in Fig.13.1, it is required to calculate and draw the speed-torque characteristics of the motor for firing angles $\alpha = 0°$, $\alpha = 45°$, and $\alpha = 90°$. The supply voltage is 60V (*rms*) , the motor armature resistance $R_a$ = 0.5 Ω and the motor voltage constant $K\emptyset = 1$ V.s / rad.

**Solution**

Using equations (13.5) , (13.6) and (13.7),

$$T_{st} = \frac{\sqrt{2}K\emptyset V_{rms}}{2\pi R_a}(1+\cos\alpha)$$

$$\omega_o = \frac{\sqrt{2}V_{rms}}{2\pi K\emptyset}(1+\cos\alpha)$$

To calculate $T_{st}$ and $\omega_o$ , the following values are obtained and drawn in Fig.13.3.

| Firing angle α | Starting torque $T_{st}$ (Nm) | No load speed $\omega_o$ (rad/s) |
|---|---|---|
| **0°** | 54 | 27 |
| **45°** | 46.1 | 23 |
| **90°** | 27 | 13.5 |

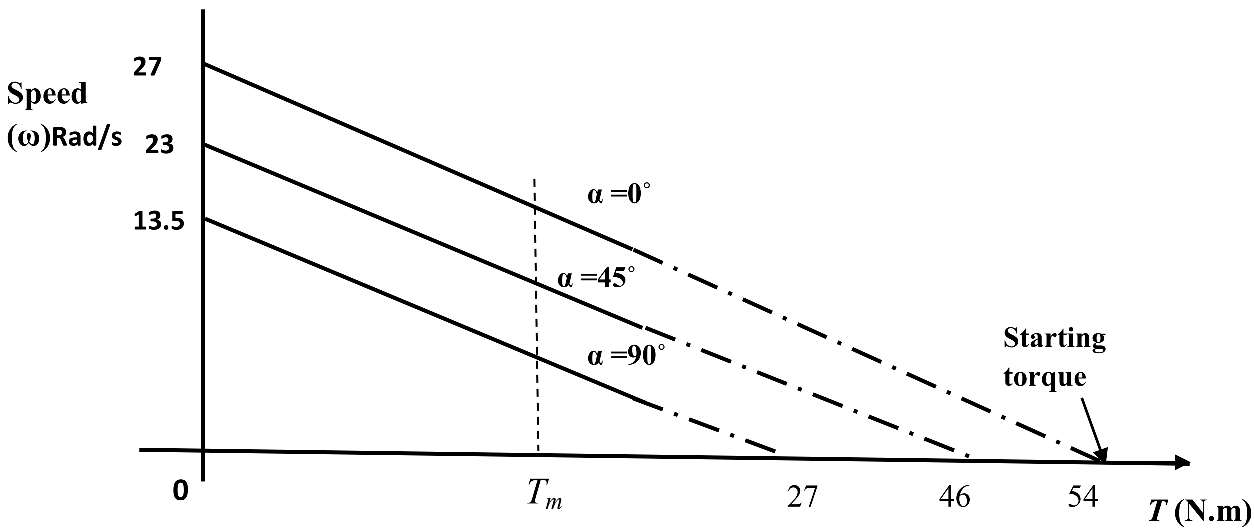



Fig.13.3 Speed-torque characteristics of a separately-excited d.c. motor controlled by single-phase half-wave rectifier drive.

### 13.2.2 Single-Phase Semiconverter with Separately-Excited d.c. Motor Load

The circuit diagram of a single-phase full-wave half-controlled (semiconverter) drive for controlling a separately-excited d.c. motor is depicted in Fig.13.4. Here a full wave rectifier bridge is supplies the field circuit, and a half-controlled bridge supplies the armature circuit. The vast majority of shunt motors are controlled in this manner.

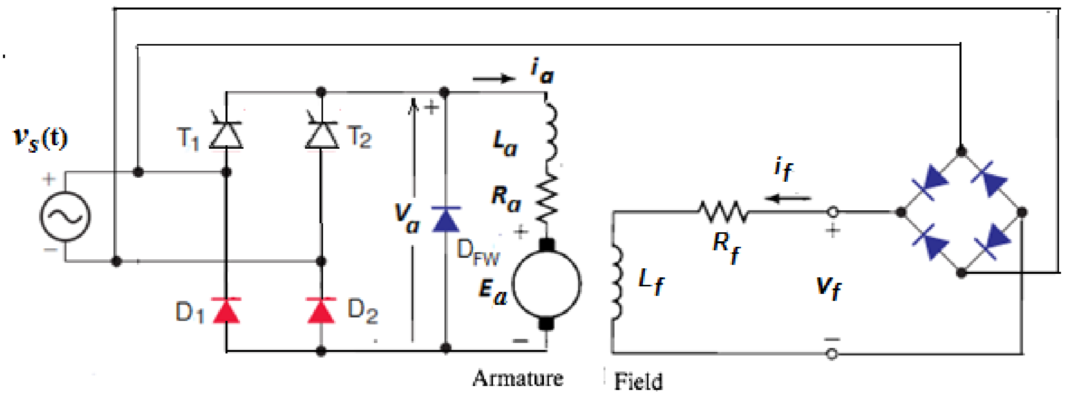



Fig.13.4 Single-phase semiconverter with d.c. motor load.

Assuming the supply voltage $v_s(t) = V_m \sin \omega t$, in the positive half-cycle, $T_1$ and $D_2$ will conduct from α to (α + δ), where α is the firing angle and δ is the conduction angle. Generally, for medium and large motors the inductance of the armature is small and hence, for the separately-excited motor, the armature current falls to zero at the instant when the back *emf* $E_a$ is equal to the supply voltage . i.e.

$$\alpha + \delta = \pi - sin^{-1}\frac{E_a}{V_m} \tag{13.8}$$

The waveforms of the voltage $v_a$ across the armature and the current $i_a$ through the armature are shown in Fig.13.5. It is obvious that the armature current is discontinuous.

**(A) Discontinuous armature current operation**

The differential equations describing the motor system, during the period the thyristors conduct, are

$$v_a = i_a R_a + L_a \frac{di_a}{dt} + e_a = i_a R_a + L_a \frac{di_a}{dt} + K\emptyset\omega_m \tag{13.9}$$

$$T_m = K\emptyset\, I_a = J\frac{d\omega_m}{dt} + B.\omega_m + T_L \tag{13.10}$$

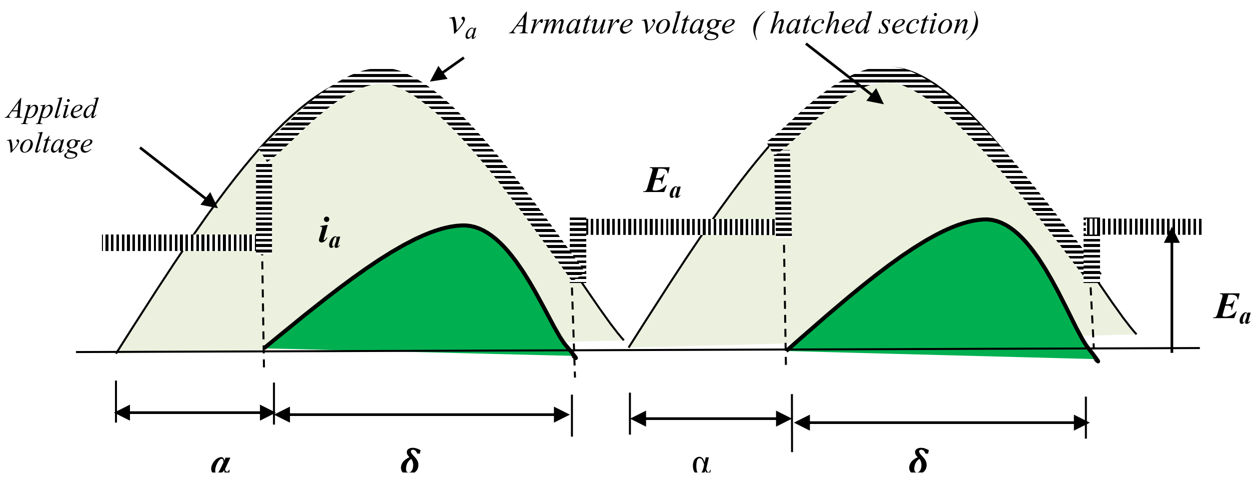



Fig.13.5 Waveforms for single-phase semiconverter operation with discontinuous armature current.

If we assume that the inertia of the rotating system is large then speed fluctuations will be negligible. If each term of $v_a$ is integrated from α to (α + δ) and then divided by π , the instantaneous voltage, current and speed will be converted to their respective average values,

$$V_{a(av)} = \frac{1}{\pi}\int_{\alpha}^{\alpha+\delta} V_m \sin\omega t \; d\omega t$$

$$= \frac{1}{\pi}\int_{\alpha}^{\alpha+\delta} L_a \frac{di_a}{dt} \; d\omega t + \frac{1}{\pi}\int_{\alpha}^{\alpha+\delta} R_a \, i_a \; d\omega t + \frac{1}{\pi}\int_{\alpha}^{\alpha+\delta} K\emptyset \, \omega_m \, d\omega t$$

Thus

$$V_{a(av)} = I_{a(av)} R_a + K\emptyset \, \omega_{m(av)} \tag{13.11}$$

where

$$V_{a(av)} = \frac{V_m}{\pi}[\cos\alpha - \cos(\alpha+\delta)] \tag{13.12}$$

and the average voltage across $L_a$ is zero.

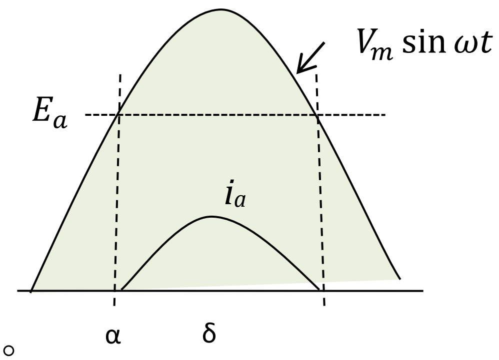
Similarly,

$$T_{m(av)} = K\emptyset \, I_{a(av)} = B.\omega_{m(av)} + T_L \tag{13.13}$$

## Example 13.2

A d.c. shunt motor , operating from a single-phase half-controlled bridge at speed of 1450 rpm, has an input voltage $v_s = 330 \sin\omega t$ and a back *emf* of 75 V. The thyristors are fired symmetrically at $\alpha = \pi / 4$ in every half-cycle, and the armature resistance is 5 Ω. Neglecting the armature inductance, calculate the average armature current and load torque.

**Solution**




$$\alpha = \pi/4 = 45^\circ$$

$$(\alpha + \delta) = \pi - sin^{-1}\left(\frac{E_a}{V_m}\right)$$

$$= \pi - sin^{-1}\left(\frac{75}{330}\right) = 166.9^\circ$$

$$\omega_{av} = \frac{2\pi n}{60} = \frac{2\pi \times 1450}{60} = 151.8 \text{ rad/s}$$

$$V_{a(av)} = \frac{1}{\pi}\int_{45^\circ}^{166.9^\circ} V_m \sin\omega t \; d\omega t = \frac{V_m}{\pi}[\cos 45^\circ - \cos 166.9^\circ]$$

$$= \frac{330}{\pi} \times 1.68 = 176.6 \text{ V}$$

$$E_a = K\,\emptyset\,\omega_{m(av)}$$

$$75 = 151.8\,K\emptyset \quad \rightarrow \quad K\emptyset = \frac{75}{151.8} = 0.494 \text{ V.s/rad}$$

Now use the general equation of the shunt motor (13.11),

$$176.6 = 5\,I_{a(av)} + 75 \qquad \therefore \quad I_{a(av)} = 20.32 \text{ A}$$

From Eq.(13.10) , $T_m = K\emptyset\, I_a$

$$\therefore T_{av} = K\emptyset\, I_{a(av)} = 0.494 \times 20.32 = 10.04 \text{ Nm}$$

The torque can also be evaluated as,

$$T_{av} = \frac{P}{\omega_{av}} = \frac{E_a I_{a(av)}}{\omega_{av}} = \frac{75 \times 20.32}{151.8} = 10.04 \text{ Nm}$$

**(B) Analysis with Continuous Armature Current Operation**

If the armature inductance is large then conduction will continue, even after the supply voltage has reversed, for which typical waveforms are shown in Fig.13.6. Hence assume continuous current operation, the average value of the armature voltage is:

$$V_{a(av)} = \frac{1}{\pi}\int_{0}^{2\pi} V_a(\omega t) d\omega t = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d\omega t$$

$$V_{a(av)} = \frac{V_m}{\pi}(1 + cos\alpha) \qquad (13.14)$$

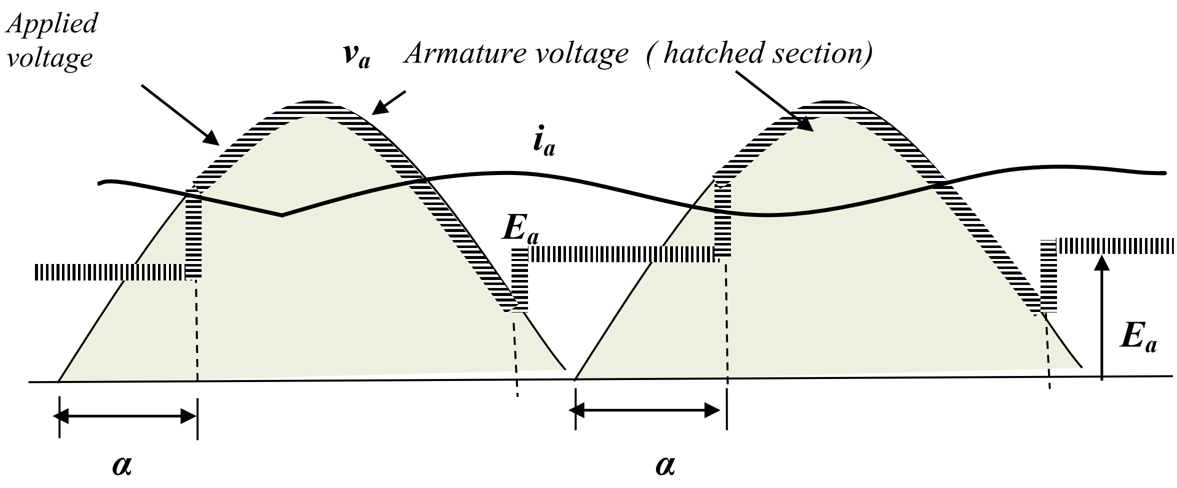



Fig.13.6 Waveforms for single-phase semiconverter operation with continuous armature current.

The average load current is:

$$I_{a(av)} = \frac{V_{a(av)} - E_a}{R_a}$$

$$I_{a(av)} = \frac{V_m}{\pi R_a}(1 + cos\alpha) - \frac{E_a}{R_a} \qquad (13.15)$$

The drive circuit shown in Fig.13.4 can also be used for the variable speed operation of a d.c. series motor. Here the motor field winding is in series with the armature and hence the armature current becomes continuous, using $D_1$ or $D_2$ as freewheeling diodes whenever the supply voltage reverses. Two modes of operation are possible. In one mode current flows through $T_1$ and $D_2$ (or $T_2$ and $D_1$) and the supply voltage appears across the motor. In the second mode $T_1$ and $D_1$ (or $T_2$ and $D_2$) conduct and the motor voltage is zero. The motor equations are:

$$v_a = V_m \sin(\omega t) \; = i_a R_T + L_T \frac{di_a}{dt} + e_a \quad \text{for } \alpha \le \omega t \le \pi \quad (13.16)$$

and

$$v_a = 0 \; = i_a R_T + L_T \frac{di_a}{dt} + e_a \quad \text{for } \pi \le \omega t \le (\pi + \alpha) \quad (13.17)$$

where $R_T$ and $L_T$ are the total resistance and inductance in the series circuit respectively.

$$T_m = K\emptyset\, I_a = J\frac{d\omega_m}{dt} + B.\omega_m + T_L \quad (13.18)$$

Integrating as before gives

$$V_{a(av)} = \frac{V_m}{\pi}(1 + cos\alpha) = R_T\, I_{a(av)} + K\emptyset\omega_{m(av)} \quad (13.19)$$

$$T_{av} = K\emptyset {I_{a(av)}}^2 = B.\omega_{m(av)} + T_L \quad (13.20)$$

For series motor, it is known that, $\emptyset = k_f\, I_{a(av)}$ , hence , $K\emptyset = Kk_f I_{a(av)}$.

Now, let $Kk_f = K_{af}$ $\rightarrow$ new constant (henry), thus the above Eq. (13.19) and Eq. (13.20) can be re-written as,

$$V_{a(av)} = \frac{V_m}{\pi}(1 + cos\alpha) = R_T\, I_{a(av)} + K_{af} I_{a(av)}\, \omega_{m(av)} \quad (13.21)$$

$$T_{av} = K_{af} {I_{a(av)}}^2 = B.\omega_{m(av)} + T_L \quad (13.22)$$

**Example 13.3**

A series d.c. motor is to be controlled by a single-phase, half-controlled, full-wave rectifier bridge as shown in Fig.13.7. The a.c. input voltage has an *rms* value of 240V at 50Hz.The combined armature and field resistance is 2.5Ω and $K_{af}$ = 300 mH. If the load torque is 30 Nm and damping is neglected, calculate the average current and the speed for α = 60˚.

**Solution**

Using Eq. (13.21),

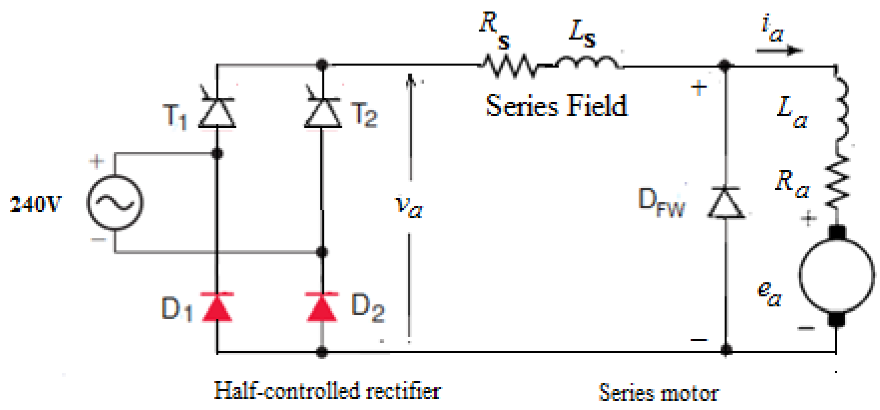



Fig.13.7 Series motor drive.

$$V_{a(av)} = \frac{V_m}{\pi}(1 + cos\alpha) = \frac{\sqrt{2}}{\pi} \times 240\ (1 + \cos 0°) = 162.11\ \text{V}$$

From Eq. (13.22) $\quad T_{av} = K_{af} I_{a(av)}{}^2 = 30 = 300 \times 10^{-3} I_{a(av)}{}^2$

$$\therefore I_{a(av)} = 10\ \text{A}$$

$$V_{a(av)} = R_T\, I_{a(av)} + K_{af} I_{a(av)}\ \omega_{m(av)}$$

$$162.11 = 2.5 \times 10 + 0.3 \times\ 10\ \omega_{m(av)}$$

$$\therefore \omega_{m(av)} = 45.7\ \text{rad /s} \longrightarrow \quad \text{n} = 436.6\ \text{rpm}$$

### 13.2.3 Single-Phase Full-Wave Fully-Controlled Rectifier Drives

The circuit diagram of a single-phase full-wave fully-controlled converter drive for controlling a separately-excited d.c. motor is depicted in Fig.13.8.

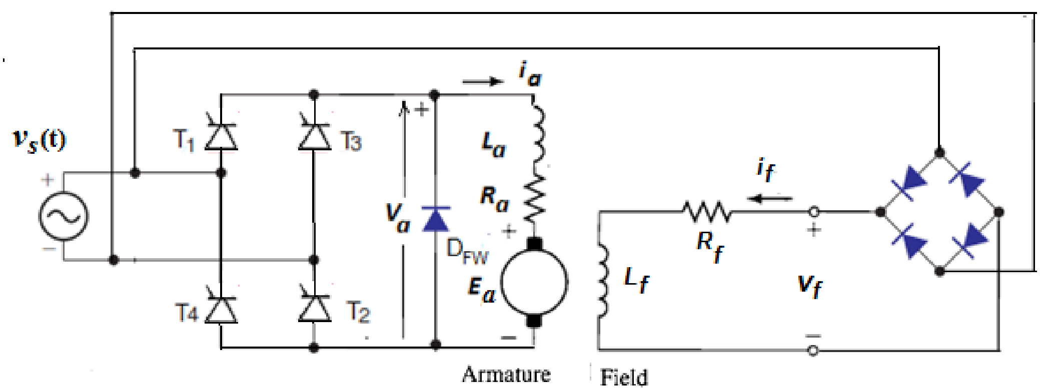



Fig. 13.8 Single-phase full-converter with d.c. motor load.

Here a full-wave rectifier bridge is supplies the field circuit, while the full-converter supplies the armature circuit. The converter has four thyristors that need alternate switching of the pairs of these thyristors $T_1$ , $T_2$ or $T_3$ , $T_4$ . The converter provides + $V_a$ or – $V_a$ depending on the value of the triggering angle $\alpha$ of the thyristors, thus two quadrant operation is possible. Armature current remains unidirectional due to the converter configuration. The vast majority of shunt motors are also controlled in this manner.

### (A) Single-phase full-converter operation with continuous motor current

The waveforms of voltage $v_a$ across the armature and the current $i_a$ through the armature are shown in Fig. 13.9 for continuous current mode of operation.

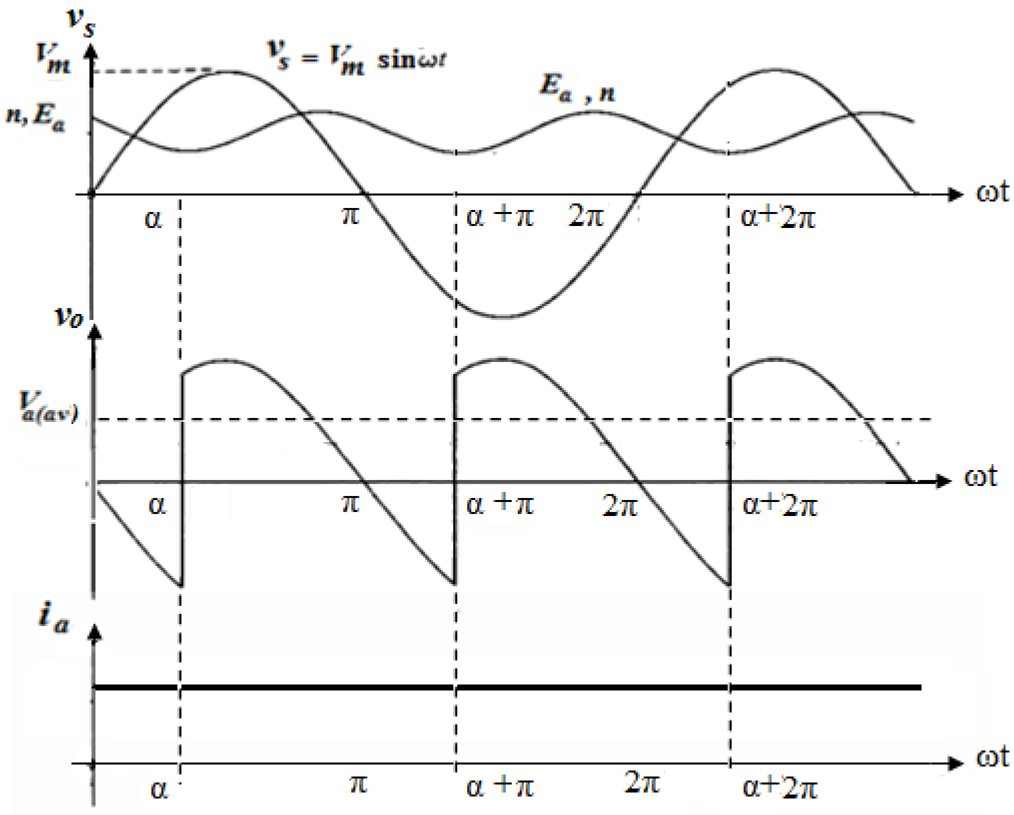



Fig.13.9 Waveforms of the armature voltage and the current for continuous current operating mode.

In any case it is obvious that the thyristor only conducts when supply voltage exceeds the back *emf*, i.e. ($V_m sin\alpha > E_a$).

When thyristors $T_1$ and $T_2$ triggered at ωt = α , $T_3$ and $T_4$ must be turned off. When thyristors $T_3$ and $T_4$ are triggered at ωt = α + π negative voltage is applied across $T_1$ and $T_2$ causes them to commutate naturally. The average value of the armature voltage, as can be deduced from Fig.13.9 is

$$V_{a(av)} = \frac{1}{2\pi}\int_{0}^{2\pi} v_a(\omega t) d\omega t = \frac{2}{2\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t)\, d\omega t$$

$$V_{a(av)} = \frac{2V_m}{\pi}\cos\alpha \qquad (13.23)$$

The average current

$$I_{a(va)} = \frac{V_{a(av)} - E_a}{R_a}$$

$$= \frac{2V_m}{\pi R_a}(cos\alpha) - \frac{E_a}{R_a} \qquad (13.24)$$

**(B) Power and power factor**

The power taken by the motor can be calculated as

$$P_{in} = P_a = I_a^2 R_a + E_a I_a \qquad (13.25)$$

$E_a I_a$ represents the output power plus the motor friction and windage losses. If the mechanical losses in the motor and electrical losses in the rectifier switches are neglected, the output power and the operating efficiency are

$$P_{out} = E_a I_a = T\omega_m \qquad (13.26)$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{E_a I_a}{P_a} \qquad (13.27)$$

Since the input current has *rms* value equal to that of the motor current, thus the operating power factor is

$$\text{Power factor } = \frac{P_a}{\frac{E_m}{\sqrt{2}}.I_L} \qquad (13.28)$$

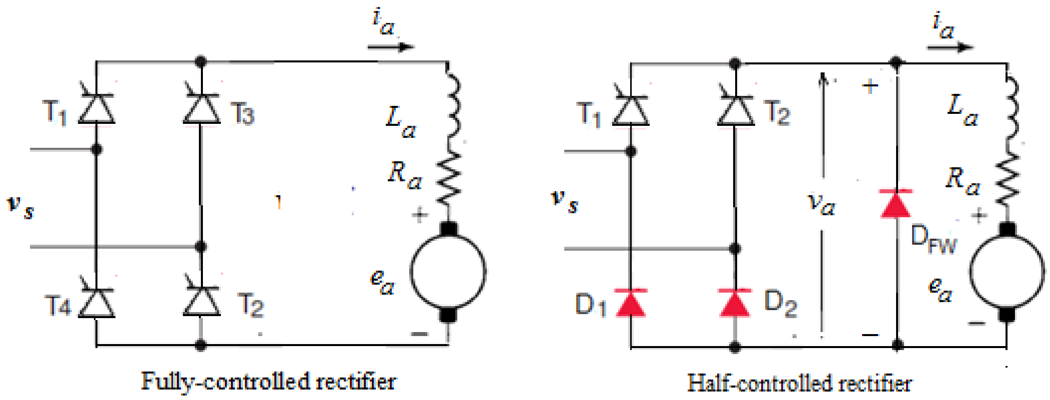
## Example 13.4

A separately-excited d.c. motor has the following parameters:

$R_a = 0.25\ \Omega$ , $K_e = 0.62$ V/ rpm.Wb , Φ (flux per pole) = 175 mWb.

The motor speed is controlled by a single-phase, full-wave bridge rectifier. The firing angle α is set at 45°, and the average speed is 1300 rpm. The applied a.c. voltage to the bridge is 230 V at 50Hz. Assuming the motor current is continuous; calculate the armature current drawn by the motor and the steady-state torque for the cases of:

(a) Fully-controlled bridge shown in Fig.13.10 (a).
(b) Half-controlled (semiconverter) bridge shown in Fig.13.10 (b).




(a) (b)

Fig.13.10.

### Solution

(a) For fully-controlled bridge with continuous current operating mode:

$$V_{a(av)} = \frac{2V_m}{\pi}\cos\alpha$$

$$V_m = \sqrt{2} \times 230 = 325.2\text{V}$$

$$V_{a(av)} = \frac{2 \times 325.2}{\pi}\cos 45° = 146.44 \text{ V}$$

$$E_a = K_e\, \varphi\, n = 0.62 \times 175 \times 10^{-3} \times 1300 = 141.05 \text{ V}$$

$$V_t = V_a = E_a + I_a R_a$$

$$\therefore I_a = \frac{V_a - E_a}{R_a} = \frac{146.44 - 141.05}{0.25} = 21.56 \text{ A}$$

Since $T_d = K_T I_a \varphi$

$K_T$ = Torque constant = 9.55 $K_e$ = 9.55 × 0.62 = 6.2

$T_d = 6.2 \times 21.56 \times 175 \times 10^{-3} = 23.4$ Nm

(b) For half-controlled (semiconverter) bridge.

$$V_{a(av)} = \frac{V_m}{\pi}(1 + cos\alpha)$$

$$V_{a(av)} = \frac{325.2}{\pi}(1 + cos45°)$$

$$V_{a(av)} = 176.78 \text{ V}$$

$E_a = K_e \varphi n = 0.62 \times 175 \times 10^{-3} \times 1300 = 141.05$V

$V_t = V_{a(av)} = E_a + I_a R_a$

$$I_a = \frac{V_{a(av)} - E_a}{R_a} = \frac{176.78 - 141.05}{0.25} = 142.95 \text{ A}$$

$T_d = 6.2 \times 142.95 \times 175 \times 10^{-3} = 155.1$ Nm

**Example 13.5**

A single-phase full converter of Fig.13.8 is used to control the speed of small separately-excited d.c. motor rated at 7.5 kW, 230 V, 1500 rpm. The converter is connected to a single-phase 230 V, 50 Hz supply. The armature resistance is $R_a$ = 0.50 Ω and the armature circuit inductance is $L_a$ = 10 mH. The motor voltage constant is $K_e$ Φ = 0.07 V/rpm. With the converter operates as a rectifier, the d.c. motor runs at 1200 rpm and carries an armature current of 35 A. Assume that the motor current is continuous, determine:

(a) The firing angle α.
(b) The power delivered to the motor.
(c) The power factor of the supply.

**Solution**

(a) The back *emf* of the motor at 1200 rpm

$E_a = k_e Ø n = 0.07 \times 1200 = 84$ V

The average armature voltage is

$V_{a(av)} = E_a + I_a R_a$

$= 84 + 35 \times 0.5 = 101.5$ V.

For the full-wave full-converter:

$$V_{a(av)} = \frac{2V_m}{\pi}\cos\alpha = \frac{2\times\sqrt{2}\times 230}{\pi}\cos\alpha = 101.5\ \text{V}$$

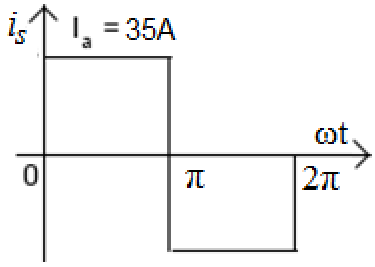
.

$$\cos\alpha = \frac{101.5\,\pi}{2\sqrt{2}\times 230} = 0.489 \qquad \text{or} \qquad \alpha = 60.65°$$

(b) The average power: $P = V_{a(av)}\, I_{a(av)} = 101.5 \times 35 = 3552.5$ W.

(c) The supply current has a square wave with amplitude $I_{a(av)} = 35$A as shown in the figure below.

$$I_s = \sqrt{\frac{1}{\pi}\int_0^{\pi}(i_a)^2 d\omega t} = I_a = 35\ \text{A}$$




The supply VA = $I_s V_s$ =230 × 35 = 8050

$$\text{power factor} = \frac{\text{Active power (W)}}{\text{apparent power (VA)}} = \frac{3552.5}{8050} = 0.441$$

**Example 13.6**

A single-phase semiconverter, shown in Fig.13.4, is used to control the speed of small separately-excited d.c. motor rated at 4.5 kW, 220 V, 1500 rpm. The converter is connected to a single-phase 220 V, 50 Hz supply. The armature resistance is $R_a = 0.50\ \Omega$ and the armature circuit inductance is $L_a = 10$ mH. The motor voltage constant is $Ke\ \Phi = 0.1$ V/ rpm. With the converter operates as a rectifier, the d.c. motor runs at 1200 rpm and carries an armature current of 16 A. Assume that the motor current is continuous and ripple-free, determine:

(a) The firing angle α.
(b) The power delivered to the motor.
(c) The supply power factor.

**Solution**

(a) The back *emf* of the motor at 1200 rpm

$E_a = k_e \varnothing n$

$= 0.1 \times 1200 = 120$ V.

$V_{a(av)} = E_a + I_{a(av)}\ R_a$
$= 120 + 16\ \times 0.5 = 120 + 8 = 128\text{V}.$

For half-controlled (semiconverter) rectifier:

$$V_{a(av)} = \frac{V_m}{\pi}(1 + \cos\alpha) = \frac{220\ \times\sqrt{2}}{\pi}(1 + \cos\alpha) = 128V$$

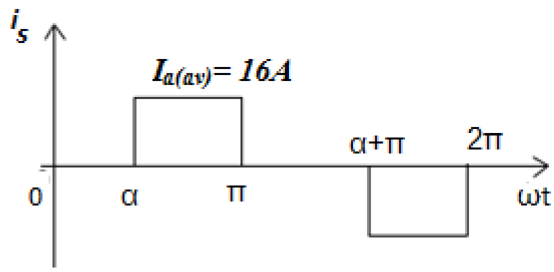
$\cos\alpha = 1.292 - 1.0 = 0.292.$

$\alpha = 73.0^{\circ}$

(b) $P = V_a I_a = 128 \times 16 = 2048$ W.

(c) The supply current is shown in the figure below

$$I_{s(rms)} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}(I_{a(av)})^2 d\omega t}$$




$$= \sqrt{\frac{(I_{a(av)})^2}{\pi}\ [\omega t]_{\alpha}^{\pi}}$$

$$= I_a\sqrt{\frac{\pi - \alpha}{\pi}} = 16 \times \sqrt{\frac{\pi - [73 \times \frac{\pi}{180}]}{\pi}} = 12.33\text{A}$$

The supply VA = 220 × 12.33 = 2712.91

$$\therefore\ PF = \frac{\text{W}}{\text{VA}} = \frac{2048}{2712.91} = 0.7549$$

**Example 13.7**

The speed of 10 hp, 230 V, 1200 rpm separately-excited d.c. motor is controlled by single-phase fully-controlled full-wave rectifier bridge. The rated armature current is 38 A, $R_a$ = 0.3 Ω, the a.c. supply voltage is 260V. The motor voltage constant is $Ke\Phi$ = 0.182V/rpm. Assume sufficient inductance is present in the armature circuit to make $I_a$ continuous and ripple-free:

(a) For $\alpha = 30°$ and rated motor current calculate,
(i) Motor torque (ii) Motor speed (iii) Supply power factor
(b) The polarity of the armature *emf* is reversed say by reversing the field excitation, calculate:
(i) the firing angle to keep the motor current at its rated value.
(ii) the power fed back to the supply.

**Solution**

(a) For $\alpha = 30°$ ,
(i) Motor torque is $T = k_e \emptyset I_a$

$K_e \emptyset = 0.182$ V/ rpm

$$= \frac{0.182}{2\pi} \times 60 = 1.74\,\mathrm{V.s/rad} \quad (K_T\emptyset = 9.55 K_e\,\emptyset)$$

Hence $T = 1.74 \times 38 = 66.12$ Nm.

$$V_{a(av)} = \frac{2\sqrt{2}}{\pi} \times V_m \cos\alpha = \frac{2\sqrt{2}\ \times 260}{\pi} \cos 30° = 202.82\ \mathrm{V}$$

(ii) $E_a = V_{av} - I_a R_a = 202.82 - 38 \times 0.3 = 191.42$ V.

$$\text{Since } E_a = K_e\,\emptyset\,n \quad \rightarrow n = \frac{E_a}{\mathrm{Ke}\ \emptyset} = \frac{191.42}{0.182} = 1051.8\ \mathrm{rpm}$$

$$\text{(iii)}\ \ PF = \frac{V_{a(av)} I_a}{V_s I_a} = \frac{V_{a(av)}}{V_s} = \frac{202.82}{260} = 0.78$$

(b) (i) $E_a = -\ 191.42$

$V_{av} = -\ 191.42 + 38 \times 0.3 = -\ 180.02$ V.

$$-\ 180.02 \ = \frac{2\sqrt{2}\ \times 260}{\pi} \cos\alpha \ \rightarrow \ \alpha = 140.24°.$$

(ii) Power fed back to the supply = $180.02 \times 38 = 6840.76$ W.

**Example 13.8**

A d.c. series motor has the following parameters:

$$R_a = 3\Omega\ , \qquad R_s = 3\Omega\ , \qquad K_{af} = 0.15\ \text{H.}$$

The motor speed is controlled by a single-phase, full-wave bridge rectifier. The firing angle is set to 45˚ and the average speed is 1450 rpm. The supply a.c. voltage to the bridge is $v_s$ = 330 sin$\omega t$ volts. Assuming continuous motor current, calculate the steady-state average motor current and torque for,

(a) Fully-controlled bridge,
(b) Half-controlled bridge.

**Solution**

(a) For fully-controlled bridge:

For continuous current operating mode, using Eq. (13.23)

$$V_{a(av)} = \frac{2V_m}{\pi} cos\alpha = \frac{2 \times 330}{\pi} \cos 45° = 148.53\ \text{V}$$

For series motor,

$$V_{a(av)} = R_T\ I_{a(av)} + K_{af} I_{a(av)}\ \omega_{m(av)}$$

$$148.53 = (3 + 3) \times I_{a(av)} + 0.15 \times\ I_{a(av)} \times \frac{2\pi \times 1450}{60}$$

$$\therefore\ I_{a(av)} = 5.16\ \text{A}$$

From Eq. (13.22) $\qquad T_{av} =\ K_{af} {I_{a(av)}}^2 = 0.15\ \times (5.16)^2 = 4\ \text{Nm}$

(b) For half-controlled bridge,

$$V_{a(av)} = \frac{V_m}{\pi}(1 + cos\alpha) = \frac{330}{\pi}(1 + \cos 45°) = 179.3\ \text{V}$$

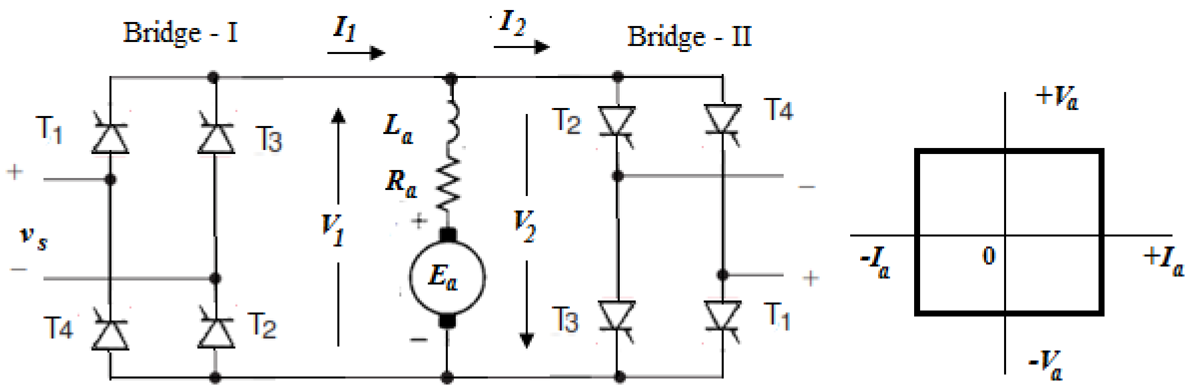
$$179.3 = (3 + 3) \times I_{a(av)} + 0.15 \times\ I_{a(av)} \times \frac{2\pi \times 1450}{60}$$

$$\therefore\ I_{a(av)} = 6.23\ \text{A}$$

From Eq. (13.22) $\qquad T_{av} =\ K_{af} {I_{a(av)}}^2 = 0.15\ \times (6.23)^2 = 5.82\ \text{Nm}$

### 13.2.4 Single-Phase Dual Converter Drives

In some industrial applications, d.c. motor may require to be operated in four quadrants without a switching changeover. In this case, duplication of power electronics converters is used. Fig.13.11 shows a simple dual converter drive circuit diagram which consists of two single-phase full bridge converters connected in inverse-parallel supplying a d.c motor. One bridge for one direction of motor current and the other bridge for the opposite direction of current. The controls are interlock to prevent their simultaneous operation to avoid short circuits on one another. Bridge-I provides operation in the first and fourth quadrants while bridge-II provides operation in second and third quadrants. Therefore, the dual converter is a four quadrant drive which allows four quadrant of machine operation without a switching changeover.




(a) (b)

Fig.13.11 Dual converter drive: (a) Circuit diagram, and (b) Quadrants of operation.

To illustrate how a speed reversal takes place, bridge-I has its firing signals removed; $i_1$ falls to zero and after few milliseconds delay, bridge-II is fired. This drive is employed for motors of rating up to 15 kW. On the circuit of Fig.13.13, positive voltages are shown by the arrowheads, though in the equations, these voltages may have negative values. These equations are:

Bridge – I operating:

$$V_1 = V_{a(av)\,1} = \frac{2V_m}{\pi}\cos\alpha_1 = V_{do}\cos\alpha_1 = E_a + I_1 R_a \qquad (13.29)$$

Bridge – II operating:

$$V_2 = V_{a(av)\,2} = -\left(\frac{2V_m}{\pi}\cos\alpha_2\right) = V_{do}\cos\alpha_2 = E_a - I_1 R_a \qquad (13.30)$$

where

$$V_{do} = \frac{2V_m}{\pi}$$

Which is the output voltage of the converter when α = 0˚.

Equations (13.29) and (13. 30) are shown as straight lines on Fig.13.12, the intersection of the machine and bridge characteristics giving the operating points.

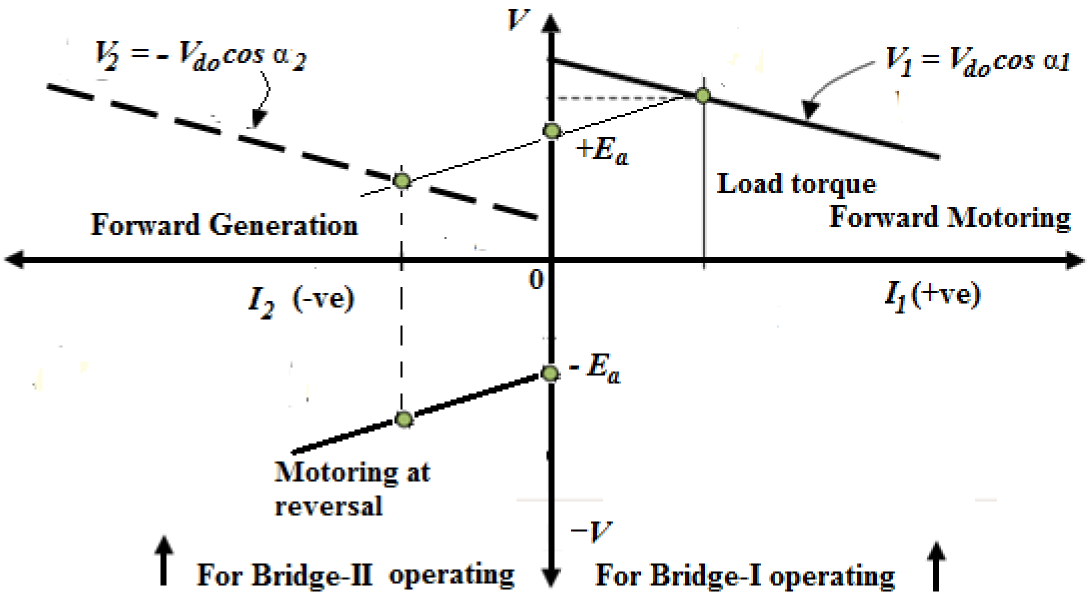



Fig.13.12 Dual converter.

## Example 13.9

A d.c. separately-excited motor rated at 10 kW, 200 V is to be controlled by dual converter. The armature circuit resistance is 0.2 Ω and the machine constant $K_e\emptyset$ is 0.35 V/ rpm. For the following conditions, determine the firing angles of the converter, the back *emf* and the machine speed given that for the converter system $V_{do}$ = 250 V. Neglect any losses in the converter circuit.

(a) Machine operates in a forward motoring mode at rated current and with terminal voltage of 200 V.

(b) Machine operates at forward generation mode at rated current and with terminal voltage of 200 V.

**Solution**

(a) For the motoring case,

$$V = V_{do}\ \cos\alpha \quad \rightarrow \quad 200 = 250\cos\alpha \quad \rightarrow \quad \cos\alpha = \frac{200}{250} = 0.8$$

$$\therefore\ \alpha = cos^{-1}(0.8) = 36.8°$$

The rated current of the machine $I_a$ = 10000 / 200 = 50 A.

$$V = E_a + I_a\ R_a \quad \rightarrow \quad 200 = E_a + 50 \times 0.2$$

$$\therefore \quad E_a = 200 - 10 = 190\ \text{V}$$

The speed of the motor can be calculated as,

$$E_a = K_e\emptyset\, n \quad \rightarrow \quad n = \frac{E_a}{K_e\emptyset} = \frac{190}{0.35} = 542.85\ \text{rpm}$$

(b) For generating mode,

$$-V = V_{do}\ \cos\alpha \quad \rightarrow \quad -200 = 250\cos\alpha$$

$$\therefore\ \alpha = cos^{-1}(-0.8) = 143.13°$$

$$E_a = V + I_a\ R_a \quad \rightarrow \quad E_a = 200 + 50 \times 0.2 = 210\ \text{V}$$

$$E_a = K_e\emptyset\, n \quad \rightarrow \quad n = \frac{E_a}{K_e\emptyset} = \frac{210}{0.35} = 600\ \text{rpm}$$

## 13.3 THREE-PHASE DC DRIVES

Three-phase converters are commonly used in adjustable speed drives from about 15 kW up to several thousand kilowatts ratings. The output voltage of a three-phase converter has less ripple contents than the single-phase converter, and therefore, the armature current will be smoother and mostly continuous.

The theory and operation of all types of the three-phase converters was fully discussed in chapter Three-Part I, for the case of passive impedance load. However, three-phase converters could be half-wave, full-wave fully-controlled, full-wave half-controlled (semiconverter) and dual converter when reversible armature current is needed. The three-phase half-wave circuit is only of theoretical importance and is generally not used in industrial applications because of the d.c. components inherent in its line currents. For medium size motors, in the range 15 – 120 kW, either the full-converters or semiconverter are used.

### 13.3.1 Three-Phase Half-Wave ( or p = *3*) Converter

In the three-phase half-wave converter, the motor load is connected between the converter positive terminal (cathodes of all thyristors) and the supply neutral as shown in Fig.13.13 .The firing angle $\alpha$ is also defined to be zero from the zero crossings of the input voltages. This converter is used for motor ratings from 10 to 50 hp and it is rarely used in practice because of the d.c. component in the line current. Waveforms of the armature (load) voltage and current are shown in Fig.13.14.

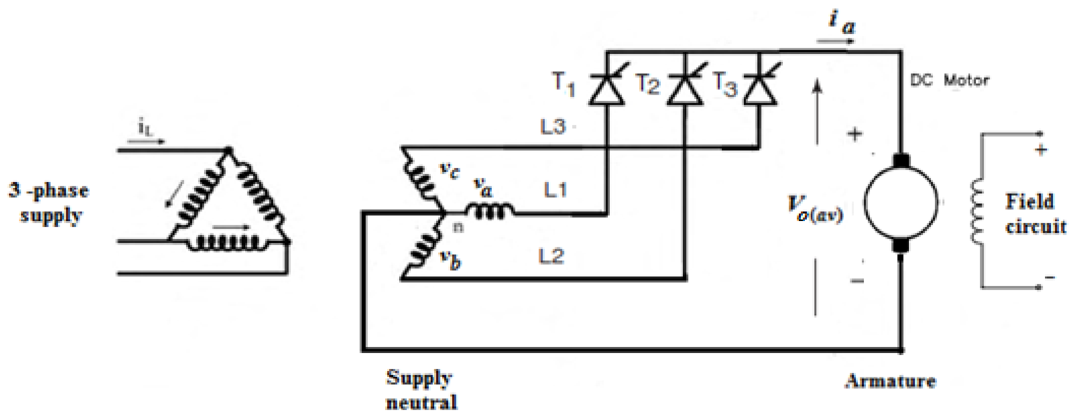



Fig.13.13 Three-phase, half-wave controlled converter with motor load.

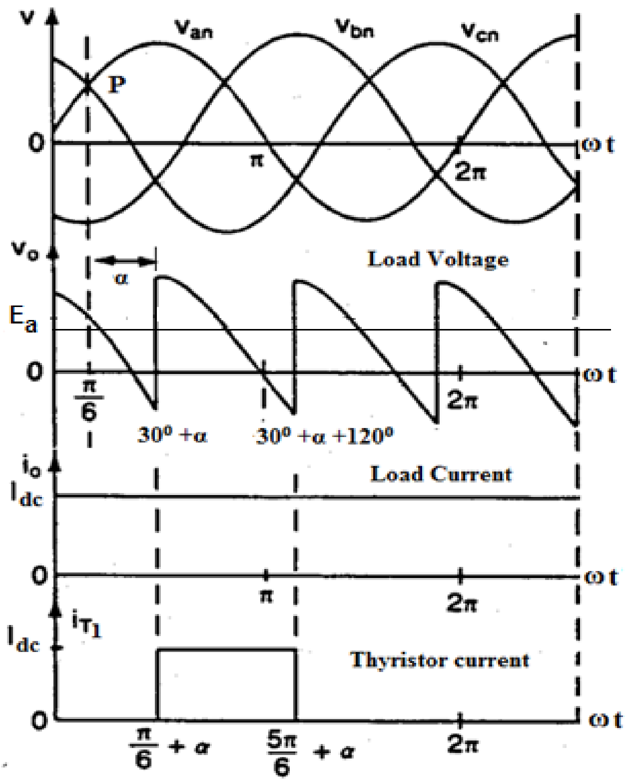



Fig.13.14.Wave forms for the three-phase half-wave converter d.c. drive.

To find the average value of the output d.c. voltage of the converter, let the transformer secondary phase to neutral voltages be,

$$v_{an} = V_m \, sin\omega t$$
$$v_{ab} = V_m \sin(\omega t - 120°)$$
$$v_{ab} = V_m \sin(\omega t - 240°)$$

Assuming continuous current conduction, the average output voltage is,

$$V_{o(av)} = \frac{1}{\frac{2\pi}{3}} \int_{30°+\alpha}^{30°+\alpha+120°} V_m \, sin\omega t \, d\omega t = \frac{3V_m}{2\pi} [-cos\omega t] \begin{matrix} 150° + \alpha \\ 30° + \alpha \end{matrix}$$

$$V_{o(av)} = \frac{3V_m}{2\pi} [-(\cos(150° + \alpha) - \cos(30° + \alpha))]$$

$$V_{o(av)} = \frac{3\sqrt{3}V_m}{2\pi} cos\alpha \qquad (13.31)$$

where $V_m$ is the peak of the supply line-neutral voltage.

A firing angle of zero degree produces the maximum output d.c. voltage for all ac-to-dc converter circuits. For continuous current conduction, each thyristor carries current for 120˚, followed by 240˚ of non-conduction. The firing angle α can be varied in the range of ±180°. For α > 90°, the output d.c. voltage becomes negative, whilst the motor current is positive and continuous. This implies operation of the converter in the fourth quadrant of the *V-I* plane as shown in Fig.13.15 where the converter operates in the inversion mode. In this mode of operation the motor supplies power to the a.c. source through the converter steadily. This mode of operation is called regenerative conversion. For example, an overhauling motor can supply its energy to the a.c. mains in this way. However, in the case of the overhauling motor, controlled braking is thus possible.

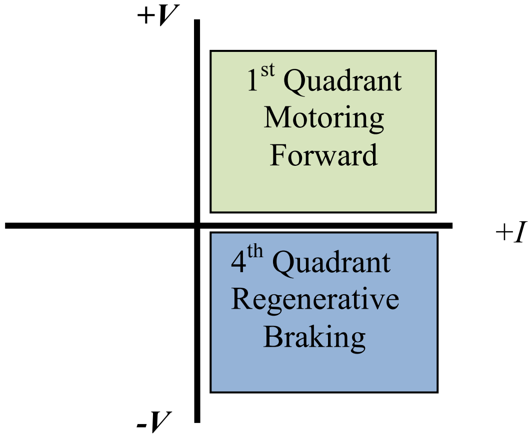



Fig.13.15 Two-quadrant operation of three-phase half-wave converter drive.

**Example 13.10**

A 100 hp, 1750 rpm, d.c. shunt motor has an armature inductance of 1.15 mH, a resistance of 0.0155 Ω and an armature voltage constant of 1.3V.s/rad. The motor is operated from a three-phase half-wave controlled-rectifier at rated armature current of 35 A. Find the firing angle α, assuming that the supply voltage is 400 V and the motor speed is 1750 rpm. Consider the thyristors to have a forward voltage drop of 1.5 Volt and assume continuous conduction.

**Solution**

Speed of the motor in rpm: $n = 1750$

Change the speed from rpm to rad/s:

$$\omega = \frac{2\,\pi n}{60} = \frac{2\,\pi}{60} \times 1750 = 183.25\ \text{rad/s}$$

Armature voltage constant $K\emptyset = 1.3$

$$\therefore \text{E}_\text{a} = K\emptyset\omega = 1.3 \times\ 183.25\ \text{V}$$

$$V_m = \sqrt{2}\ \times \frac{400}{\sqrt{3}} = 326.2\ \ \text{V}$$

$$V_{o(av)} = \frac{3\sqrt{3}V_m}{2\pi} cos\alpha = \frac{3\sqrt{3} \times 326.2}{2\pi} cos\alpha = \ 270.08 \cos \alpha$$

$$V_{o(av)} = E_a + I_a\ \ R_a +\ V_T\ \leftarrow\ \text{Thyristor drop}$$

$$270.08 \cos \alpha\ \ = 238.22 +\ 15.5 \times\ 10^{-3}\ \times\ 35\ +\ 1.5\ = 240.26$$

$$\therefore\ \alpha = \cos^{-1} \frac{240.26}{270.08} = \cos^{-1} 0.8895 = 27.2°$$

### 13.3.2 Three-Phase Semiconverter Drive

The three-phase semiconverter is a one-quadrant drive. Its circuit includes a freewheeling diode $D_{FW}$ to maintaining continuous load current. It uses three thyristors and three diodes; hence a cost advantage is obtained compared with the full-converter. The circuit diagram for a separately-excited d.c. motor supplied from a three-phase a.c. supply through a three-phase semiconverter is shown in Fig.13.16. This converter

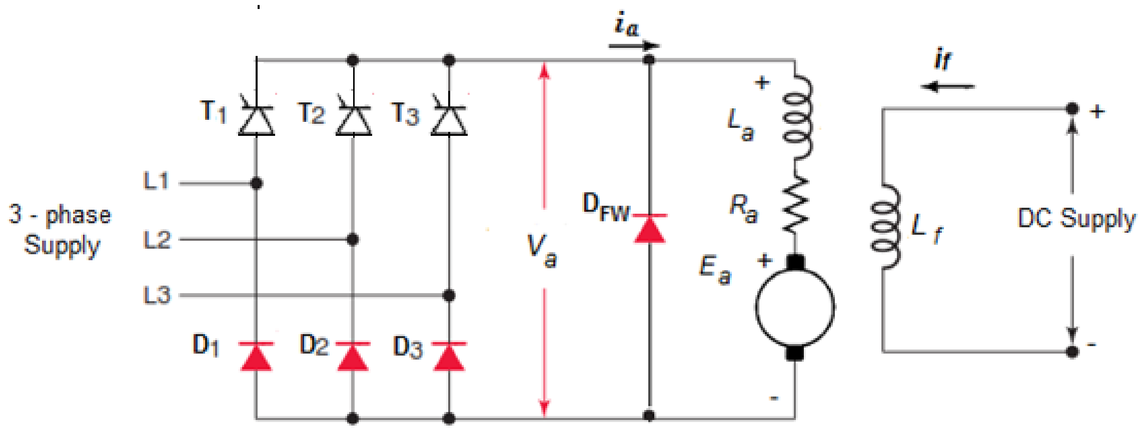


Fig.13.16 Three-phase semiconverter drive.

is used for motor ratings from 15 to 150 hp. The field converter may be single-phase or three-phase semiconverter with firing angle of $\alpha_f$.

Assuming continuous current operation, the average value of the armature voltage at the motor terminals is a contribution from the upper half-bridge plus a contribution from the uncontrolled lower bridge. Hence for all firing angles we can write:

$$V_{o(av)} = \frac{3\sqrt{3}}{2\pi} V_m \, cos\alpha + \frac{3\sqrt{3}}{2\pi} V_m$$

$$= \frac{3\sqrt{3}}{2\pi} V_m(1 + \cos\alpha) \tag{13.32}$$

The average armature current is:

$$I_{a(va)} = \frac{V_{a(av)} - E_a}{R_a} \tag{13.33}$$

$$= \frac{3\sqrt{3}V_m}{2\pi R_a}(1 + cos\alpha) - \frac{E_a}{R_a} \tag{13.34}$$

For discontinuous current operation, the above equations are not valid.

**Features:**

- Since only three thyristors are used, the circuit is not expensive and a simple control circuitry is required.
- Dynamic braking can be performed by switching armature connection to an external resistance.
- Operation is in the first quadrant only (Fig.13.17). However, bi-directional rotation can be obtained by reversing field current or armature terminals when the motor has been stopped.

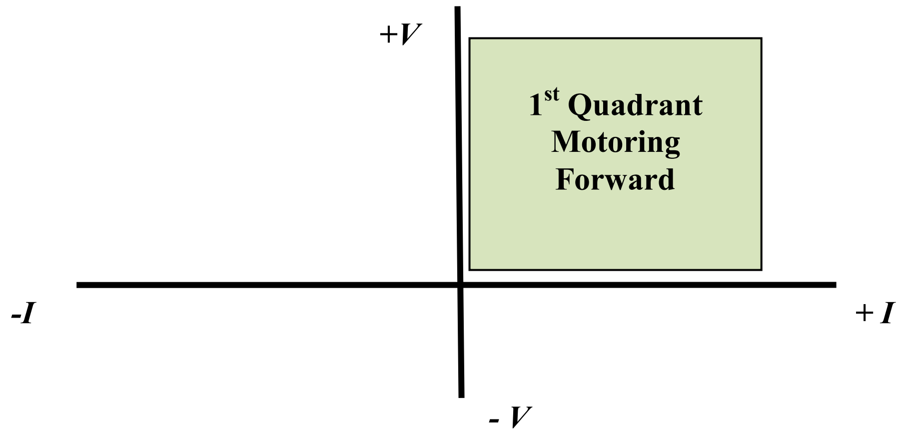



Fig.13.17 One-quadrant operation of the three-phase semiconverter drive.

**Example-13.11**

A Three-phase half-controlled thyristor bridge with 400 V, three-phase, 50 Hz supply is feeding a separately-excited d.c. motor. Armature resistance is 0.2 Ω, armature rated current is 100 A and back *emf* constant is 0.25 V/rpm. Determine the no-load speed if the no-load armature current is 5 A and firing angle is 45˚. Also determine the firing angle to obtain a speed of 1500 rpm at rated current.

**Solution**

Armature voltage at no-load,

$$V_{o(o)} = \frac{3\sqrt{3}}{2\pi} V_m(1+\cos\alpha)$$

$$= \frac{3\sqrt{3}}{2\pi} \times \frac{400\sqrt{2}}{\sqrt{3}}(1+cos45°) = 461 \text{ V}$$

Back *emf* at no-load , $E_{ao} = V_{a(o)} - I_{ao}R_a = 461 - 5 \times 0.2 = 460$ V

Speed at no-load

$$n_o = \frac{E_{ao}}{K_e\emptyset} = \frac{460}{0.25} = 1840 \text{ rpm}$$

Back *emf* developed at speed of 1500 rpm

$$E_{ao} = 1500 \times 0.25 = 375 \text{ V}$$

Armature voltage will be:

$$V_o = E_a + I_a R_a = 375 + 100 \times 0.2 = 395 \text{ V}$$

Also

$$V_o = \frac{3\sqrt{3}}{2\pi} V_m (1 + \cos\alpha)$$

$$395 = \frac{3\sqrt{3}}{2\pi} \times \frac{400\sqrt{2}}{\sqrt{3}} (1 + \cos\alpha)$$

$$\alpha = 62.45°$$

### 13.3.3 Three-Phase Full-Converter Drive

The circuit diagram for a separately-excited d.c. motor supplied from a three-phase a.c. supply through a three-phase full-converter is shown in Fig.13.18.

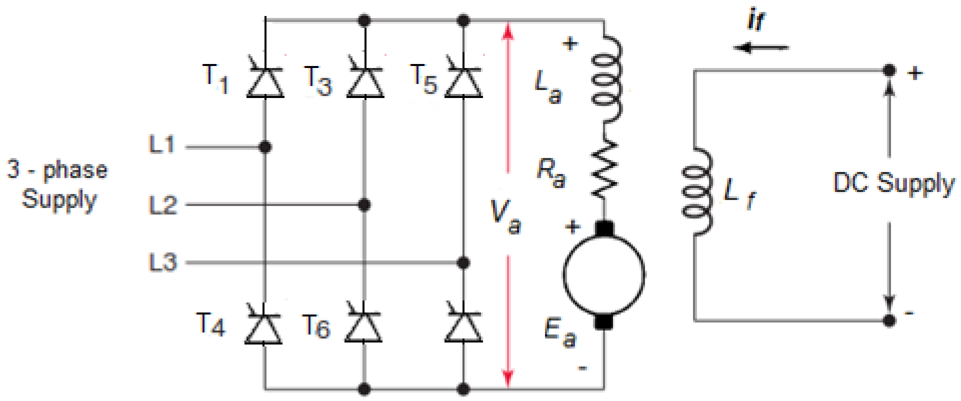



Fig.13.18 Three-phase full-converter with a separately-excited d.c. motor load.

If the motor armature inductance is large and the firing angle is small then the armature current is likely to be continuous. However, with small armature inductance and large firing angles the armature current may become discontinuous particularly when the back *emf* is relatively high.

- For continuous current operation, the armature voltage has an average value which is the same as that given in Eq.(3.40) in Chapter Three - Part I , repeated here,

$$V_{a(av)} = \frac{3\sqrt{3}}{\pi} V_m \cos\alpha \qquad (13.35)$$

Thus, the average armature current is:

$$I_{a(va)} = \frac{V_{a(av)} - E_a}{R_a} \tag{13.36}$$

$$= \frac{3\sqrt{3}V_m}{\pi R_a}\cos\alpha - \frac{E_a}{R_a} \tag{13.37}$$

- For discontinuous current operation of the full-converter, the waveforms of the voltage and currents are shown in Fig.13.19.

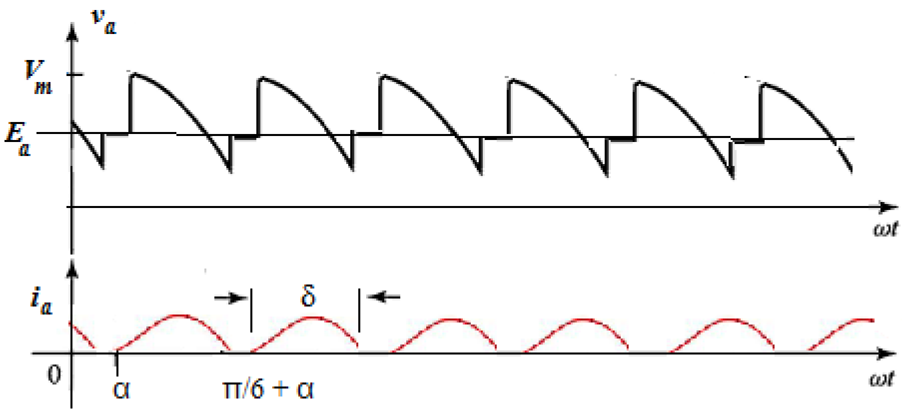



Fig.13.19 Discontinuous current operation waveforms of the full-converter.

The differential equations describing the motor system, during the period the thyristors conduct, are

$$v_a = i_a R_a + L_a \frac{di_a}{dt} + e_a \tag{13.38}$$

$$\sqrt{3}V_m \sin(\omega t + 30°) = i_a R_a + L_a \frac{di_a}{dt} + K\emptyset\omega_m$$

$$T_m = K\emptyset\, I_a = J\frac{d\omega_m}{dt} + B.\omega_m + T_L \tag{13.39}$$

If it is assumed that the inertia of the rotating system is large then speed fluctuations will be negligible. If each term of $v_a$ is integrated from α to (α + π/6) and then divided by π/3, the instantaneous voltage, current and speed will be converted to their respective average values,

$$V_{a(av)} = \frac{3}{\pi}\int_{\alpha}^{\alpha+\pi/6} \sqrt{3}V_m\,(\sin\omega t + 30°)\; d\omega t$$

$$= \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi/6} L_a \frac{di_a}{dt}\, d\omega t + \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi/6} R_a\, i_a\, d\omega t + \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi/6} K\emptyset\, \omega_m\, d\omega t$$

Thus

$$V_{a(av)} = I_{a(av)} R_a + K\emptyset\, \omega_{m(av)} \quad (13.40)$$

where

$$V_{a(av)} = \frac{3\sqrt{3}V_m}{\pi}[\cos(\alpha + 30°) - \cos(\alpha + 90°)] \quad (13.41)$$

and the average voltage across $L_a$ is zero.

Similarly,

$$T_{m(av)} = K\emptyset\, I_{a(av)} = B \cdot \omega_{m(av)} + T_L \quad (13.42)$$

### 13.3.4 Three-Phase Dual Converter Drive

Four-quadrant operation of a medium and large size d.c. motor drive (200-2000 hp) can be obtained by the three-phase dual converter shown in Fig.13.20. The average motor voltage is required to be equal for both converters, which required that the firing angles of the two sets of the thyristors should sum to 180˚.

The armature voltage supplied by converter-1 (for continuous current operation) is

Bridge – I operating:

$$V_1 = V_{a(av)1} = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha_1$$

$$= V_{do}\cos\alpha_1 = E_a + I_1 R_a \quad (13.43)$$

Bridge – II operating:

$$V_2 = V_{a(av)2} = -\left(\frac{3\sqrt{3}V_m}{\pi}\cos\alpha_2\right)$$

$$= V_{do}\cos\alpha_2 = E_a - I_1 R_a \quad (13.44)$$

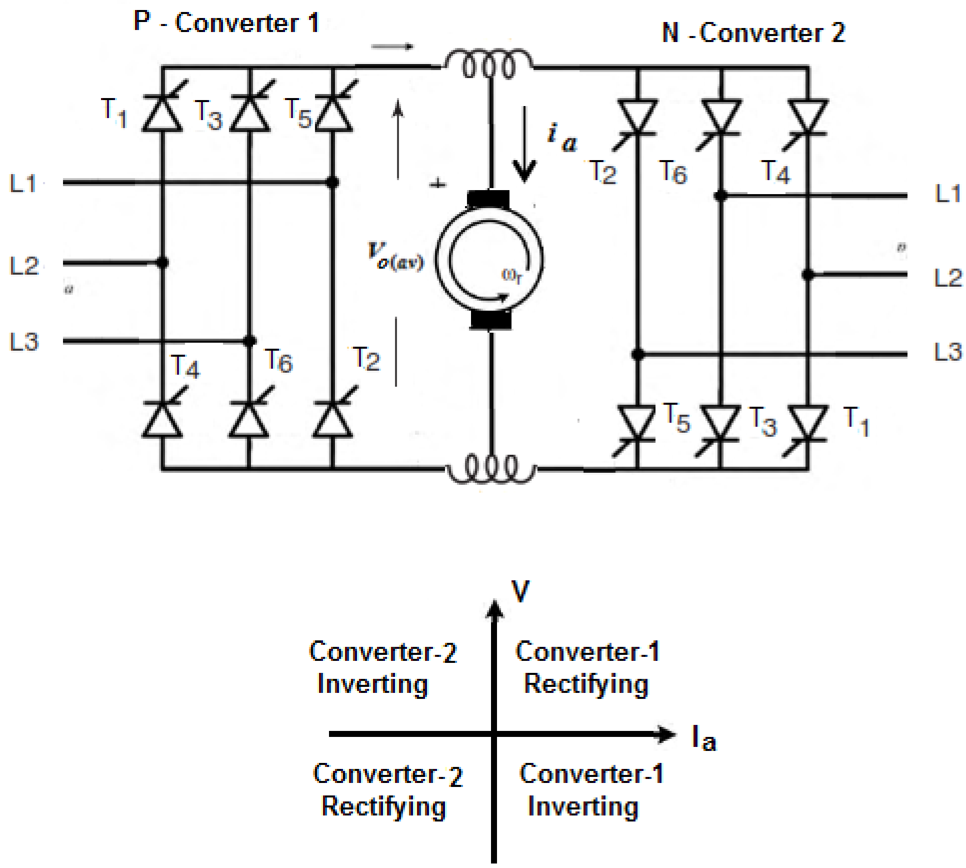



Fig.13.20 Four-quadrant three-phase d.c. drive.

where $$V_{do} = \frac{3\sqrt{3}V_m}{\pi}$$

and $\alpha_2 = \pi - \alpha_1$ .

Two modes of operation can be achieved with this circuit:

**(a) Circulating current operating mode:**

Here, instantaneous values of circulating current are limited by use of reactors and mean level is controlled by current loop. Circulating current may be constant giving linear characteristic or it may be reduced to zero giving higher gain portion of overall characteristic.

Advantage: Continuous bridge current maintain armature current at all times, no discontinuity occurs.

Disadvantage: Presence of circulating current reduces efficiency.

**(b) Circulating current-free operation mode:**

In this mode only one converter operates at a time. Logic used to prevent the two bridges being turn on at the same time. Reactors or inductors used to maintain continuous current down to acceptable low levels. Discontinuity occurs at zero and also a time delay (*ms*) introduced at the zero current level.

Advantage: Higher efficiency than circulating current schemes, hence used more widely.

Disadvantage: Dead time, discontinuity in zero current regions.

**Example 13.12**

A three-phase full-converter, shown in Fig.13.21, is used to control the speed of a separately-excited d.c. motor rated at 100 kW, 600 V, 2000 rpm. The converter is connected to a three-phase 400 V, 50 Hz supply.

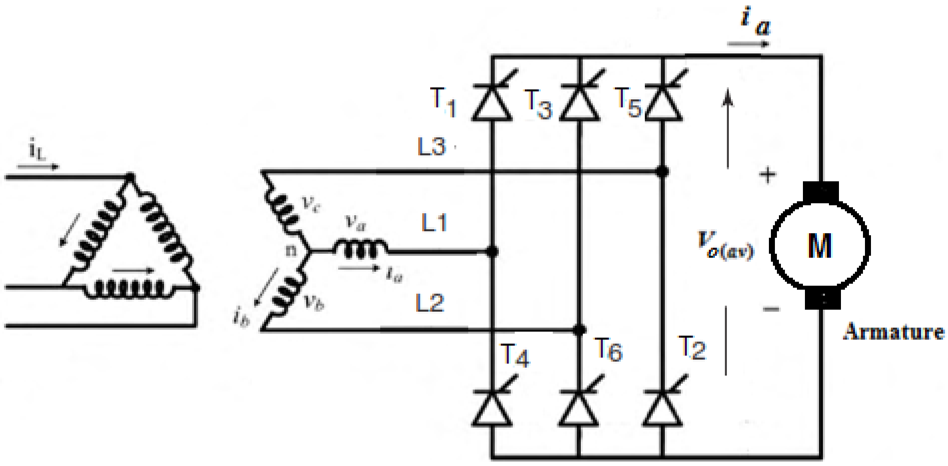



Fig.13.21 Three-phase full-converter d.c. drive.

The armature resistance $R_a$ = 0.051 Ω and the armature circuit inductance is $L_a$ = 10mH. The motor voltage constant is $K_e\Phi$ = 0.25 V/rpm. The rated armature current is 100 A and the no-load current is 10 A.
With the converter operates as a rectifier, and assuming that the motor current is continuous and ripple-free, determine:

(a)The no load speed when the firing angles: $\alpha = 0^0$ and $\alpha = 60^0$.
(b)The firing angle to obtain the rated speed of 2000 rpm at rated motor current.

**Solution**

(a) At no-load condition

$$V_m = \frac{400}{\sqrt{3}} \times \sqrt{2} = 325.22 \text{ V}$$

Let the converter output voltage = $V_{o(av)}$ = armature terminal voltage $V_a$ :

$$V_{o(av)} = \frac{3\sqrt{3}}{\pi} V_m \cos\alpha$$

$$V_{o(av)} = \frac{3\sqrt{3}}{\pi} \times 325.22 \cos\alpha$$

$V_{o(av)} = 538 \cos\alpha$

For $\alpha = 0^0$ , $\longrightarrow$ $V_{o(av)} = 538$ V

$E_a = V_{o(av)} - I_a R_a = 538 - 10 \times 0.051 = 537.5$ V

No-load speed:

since $E_a = K_e \varphi n$

Hence

$$n_o = \frac{E_a}{K_e \emptyset} = \frac{537.5}{0.25} = 2145 \text{ rpm}$$

For $\alpha = 60^0$ :

$V_{o(av)} = 538 \cos 60^0 = 269$ V

$E_a = V_{a(av)} - I_a R_a = 269 - 10 \times 0.051 = 268.4$ V

$$n_o = \frac{E_a}{K_e \emptyset} = \frac{268.41}{0.25} = 1073.64 \text{ rpm}$$

(b) At full-load condition

$E_a = K_e \varphi n = 0.25 \times 2000 = 500$ V

$V_t = E_a + I_a R_a = 500 + 100 \times 0.051 = 505$ V

$505 = 538 \cos\alpha$

Hence $\alpha = 20.14^0$

## PROBLEMS

**13.1** A separately-excited d.c. motor is supplied via a half-control full-wave single-phase rectifier bridge. The supply voltage is 240 V(*rms*), the thyristors are fired at angle of 110 ° and the armature current continues for 50 ° beyond the voltage zero. Determined the motor speed for a torque of 1.8 Nm, given the motor torque characteristics is 1.0 Nm /A and its armature resistance is 6 Ω. Neglect all rectifier losses.

[Ans: 864 rpm.]

**13.2** The speed of an 8 kW, 220V, 1250 rpm separately-excited d.c. motor is controlled by a single-phase full converter as shown in Fig.13.22. The rated armature current is 30 A. The armature resistance is $R_a = 0.25\ \Omega$ and armature inductance is $L_a = 15$ mH. The a.c. supply voltage is 300 V. The motor voltage constant is $K_e\,\varphi = 0.20$ V/rpm. Assume that motor current is continuous and ripple free. For firing angle α = 60° and rated motor current, determine: (a) Speed of the motor, (b) Motor torque, and (c) The input power to the motor.

[Ans : (a) 637.5 rpm , (b) 57.33 Nm , (c) 4050 W]

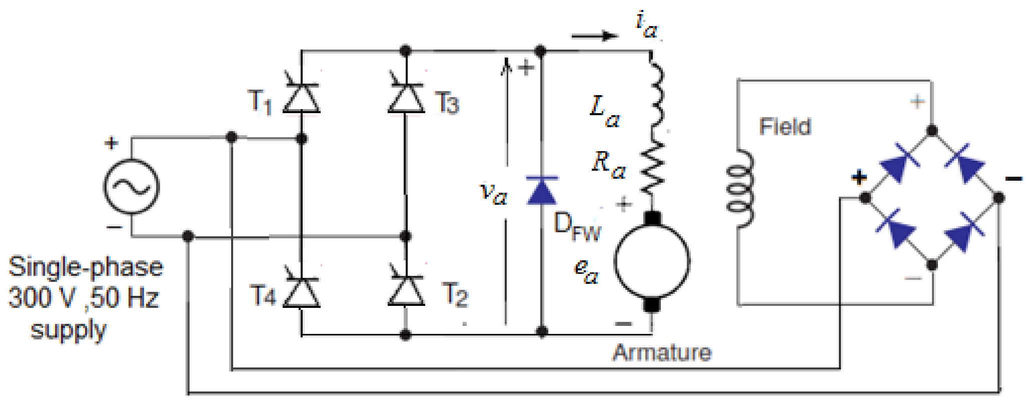



Fig.13.22.

**13.3** A single-phase source rated at 230 V, 50 Hz supplies power to a shunt-excited d.c. motor via a full-wave fully-controlled rectifier. The armature resistance $R_a = 1.5\ \Omega$ and the armature inductance $L_a = 25$ mH. The motor voltage constant $K_e\,\phi = 0.182$ V/rpm. At certain speed of operation the back *emf* is 85.6 V. When the thyristor firing angle is 45°, calculate the average value of the armature current and the power delivered by the motor. Assume that sufficient inductance is present in the armature circuit to make $I_a$ continuous and ripple free.

[Ans: 40.56 A, 3472 W]

**13.4** A separately-excited d.c. motor is rated at 8 kW, 230 V, 1200 rpm is supplied with power from a 6-pulse three-phase fully-controlled bridge rectifier. The armature circuit resistance is 0.50 Ω and the machine constant $K\emptyset$ is 1.5 V.s/rad. The motor is assumed to operate in continuous current mode, calculate for firing angles of $\alpha = 0^\circ$ and $\alpha = 30^\circ$, (a) the motor speed in rpm , (b) the power factor , and (c) the efficiency of the system. Neglect any losses in the converter circuit and assume that the load torque remains constant for each case.

[Ans : For $\alpha = 0^\circ$ : (a) 1181.14 rpm, (b) $PF = 0.80$ , (c) $\eta = 89.5\%$.
For $\alpha = 30^\circ$: (a) 1023.6 rpm, (b) $PF = 0.792$, (c) $\eta = 88.16\%$.]

**13.5** A d.c. shunt motor operating from a single-phase half control bridge at a speed of 1450 rpm has an input voltage $v_s = 330 \sin\omega t$ and a back *emf* of 75 V. The SCRs are fired symmetrically at ($\alpha = \pi/4$) in every half cycle and the armature resistance is (5 Ω). Neglecting the armature inductance, calculate the average armature current and load torque.

[Ans: $I_a = 20.32$ V, $T = 10.04$ Nm]

**13.6** A series d.c. motor is supplied from a bridge rectifier, the a.c. input of which has *rms* value of 230 V, 50 Hz. The combined armature and field resistance is 2 Ω and the field constant $k'_f$ of the motor is 0.23 H. If the load torque is 20 Nm and the damping can be neglected, calculate the average current and speed.

[Ans: $I_a = 9.33$ A, n = 838.5 rpm]

*(Hint: in series motor $E_a = k'_f\, i\, \omega$ , $\omega$ = speed ( rad/s))

**13.7** A 500 V, 1500 rpm d.c. motor is connected to a 400 V, three-phase, 50 Hz line using a three-phase fully-controlled bridge rectifier. The full-load armature current is 1500 A and the armature resistance is 0.05 Ω. Calculate:

(a) The required firing angle α under rated full load conditions.
(b) The firing angle required so that the motor develops its rated torque at 600 rpm.

[Ans: (a) $\alpha = 22.3^o$, $\alpha = 63^o$]

**13.8** The speed of a 120 kW, 600 V, 1800 rpm, separately-excited d.c. motor is controlled by a three-phase fully-controlled full-converter (6-pulse converter) as shown in Fig.13.23. The converter is operating from a three-phase 400 V, 50 Hz supply. The rated armature current of the motor is 115 A. The motor parameters are:

$R_a = 0.08\ \Omega$ , $L_a = 7.5$ mH , $K_e\Phi = 0.278$ V/rpm

(a) Find the no-load speeds at firing angles $\alpha = 0°$, and $\alpha = 45°$. Assume that, at no-load, the armature current is 10% of the rated current and is continuous.
(b) Find the firing angle to obtain the rated speed of 1800 rpm at rated motor current.

[ Ans: (a) For $\alpha = 0°$, $n_o = 1900$ rpm. For $\alpha = 45°$, $n_o = 1335$ rpm, (b) $\alpha = 24°$]

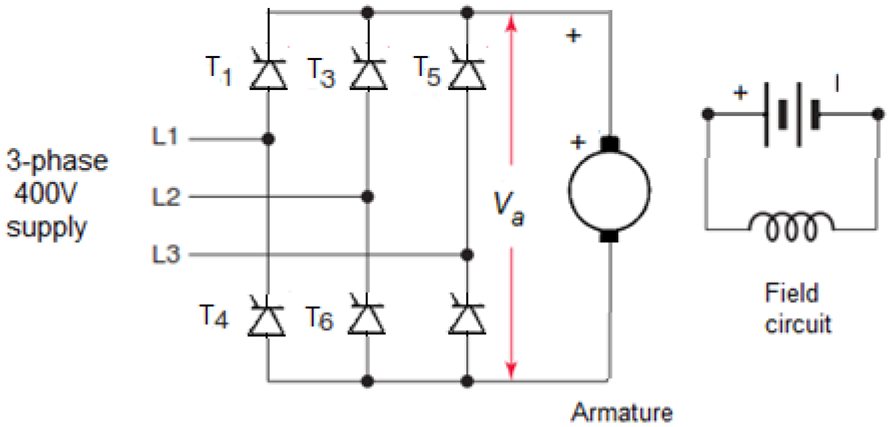



Fig.13.23.

**13.9** A 300 V, 1500 rpm d.c. motor is connected to a 400V, three-phase, 50 Hz line using a three-phase fully-controlled bridge rectifier. The full-load armature current is 2000 A and the armature resistance is 0.005 Ω. Calculate:

(a) The required firing angle α under rated full load conditions.
(b) The firing angle required so that the motor develops its rated torque at 500 rpm.

[ Ans: 56˚, 76˚]

**13.10** A 10 kW , 300 V , 1000 rpm d.c. shunt motor has an armature resistance of $19.4 \times 10^{-3}$ Ω , armature inductance of 1.1 mH , and a voltage constant of 1.38 V.s/rad. The motor is operated from a fully-controlled, three-phase rectifier. Find the firing angle α in order that the motor is operated at speed of 860 rpm , assuming that the supply voltage is 220V , consider the thyristors to have a forward voltage drop of 1.0 volt and assume continuous current operating mode. Also calculate the efficiency of the drive if the load torque is constant.

[Ans: $\alpha = 30°$, $\eta = 97$ %]

**13.11** A separately-excited d.c. motor rated at 10 kW, 400 V, 1000 rpm is supplied with power from a fully-controlled, three-phase bridge rectifier. The supply to the rectifier is assumed ideal and rated at 220 V, 50 Hz. The motor has an armature resistance of 0.2 Ω and sufficient added inductance to maintain continuous armature current condition. The motor has voltage constant of 1.38 V.s. rad$^{-1}$, and it delivers rated power at zero firing angle. If the firing angle is retarded to 30˚ calculate the speed, power factor and efficiency of operation if the load torque remains constant.

[Ans: $n$ = 866 rpm, $PF$ = 0.48, $\eta$ = 97.3 %]

**13.12** A 600 V, 50 A d.c. separately-excited motor is fed form a three-phase half-controlled rectifier. The armature resistance of the motor is 0.1 Ω, the armature inductance is 40 mH and the machine constant $K_e\ \phi$ is 0.3 V/rpm. The rectifier is fed form 400 V, 50 Hz three-phase supply. It is required to calculate:

(a) The no-load speed when the firing angle is 30° and the no-load current is 5 A.

(b) The firing angle to obtain a speed of 1600 rpm.

[Ans: (a) 1678 rpm, (b) 37.26°]

**13.13** The speed of 75 kW, 600 V, 2000 rpm separately-excited d.c. motor is controlled by a three-phase fully-controlled full-wave rectifier bridge. The rated armature current is 132 A, $R_a$ = 0.15 Ω, and $L_a$ = 15 mH. The converter is operated from a three-phase, 415 V, 50 Hz supply. The motor voltage constant is $K_e\Phi$ = 0.25 V/rpm. Assume sufficient inductance is present in the armature circuit to make $I_a$ continuous and ripple-free:

(a) With the converter operates in rectifying mode, and the machine operates as a motor drawing rated current, determine the value of the firing angle α such that the motor runs at speed of 1400 rpm.

(b) With the converter operates in inverting mode, and the machine operates in regenerative braking mode with speed of 900 rpm and drawing rated current, calculate the firing angle α.

[Ans: (a) 48.72° , (b) 111.47° ]

**13.14** A separately-excited d.c. motor rated at 55 kW, 500 V, 3000 rpm is supplied with power from a fully-controlled, three-phase bridge rectifier. The bridge is supplied from a three-phase source rated at 400 V, 50 Hz. The motor has an armature resistance of 0.23 Ω. Series inductance is present in the armature circuit to make the current continuous. Speed adjustment is required in the range 2000-3000 rpm while delivering rated torque (at rated current). Calculate the required range of the firing angles.

(Hint: The output power of the motor = $E_aI_a = T\omega$)

[Ans: 0° < α < 20.3° ]

**13.15** A separately-excited d.c. motor rated at 30 kW, 600 V, 2000 rpm with armature resistance $R_a$= 0.5 Ω and inductance $L_a$ = 30 mH. The motor driving a load whose torque is directly proportional to the speed. The speed of the motor is to be controlled from rated speed to one half speed using a three-phase, fully-controlled bridge rectifier. The rectifier is supplied from a three-phase source of 400 V, 50 Hz. Determine the range of thyristor firing angles required if an additional sufficient inductance is inserted to maintain continuous conduction. The Torque-speed character-ristics of the drive is shown in Fig.13.24.

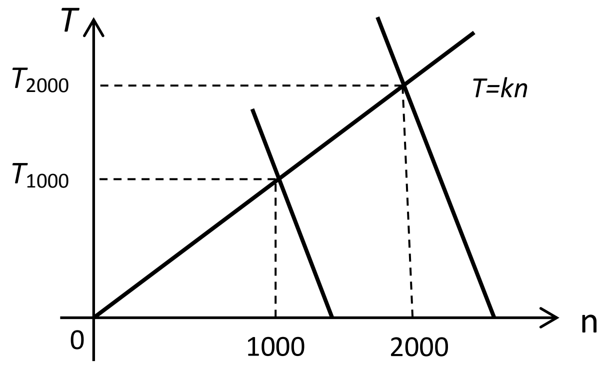



Fig.13.24 Torque-speed characteristics of the drive in problem 13.15.

[Ans : $0^o < \alpha < 69.2^o$ ]